The dark flow induced small scale kinetic Sunyaev Zel'dovich effect

Pengjie Zhang[1]

[1] *Key Laboratory for Research in Galaxies and Cosmology, Shanghai Astronomical Observatory, Nandan Road 80, Shanghai, 200030, China;*
pjzhang@shao.ac.cn

7 July 2010

ABSTRACT
Recently Kashlinsky et al. (2008, 2010) reported a discovery of a $\sim 10^3$ km/s bulk flow of the universe out to $z \simeq 0.3$, through the dark flow induced CMB dipole in directions of clusters. We point out that, if this dark flow exists, it will also induce observable CMB temperature fluctuations at multipole $\ell \sim 10^3$, through modulation of the inhomogeneous electron distribution on the uniform dark flow. The induced *small scale* kinetic Sunyaev Zel'dovich (SZ) effect will reach $\sim 1\mu K^2$ at multipole $10^3 \lesssim \ell \lesssim 10^4$, only a factor of ~ 2 smaller than the conventional kinetic SZ effect. Furthermore, it will be correlated with the large scale structure (LSS) and its correlation with 2MASS galaxy distribution reaches $0.3\mu K$ at $\ell = 10^3$, under a directional dependent optimal weighting scheme. We estimate that, WMAP plus 2MASS should already be able to detect this dark flow induced small scale kinetic SZ effect with $\sim 6\sigma$ confidence. Deeper galaxy surveys such as SDSS can further improve the measurement. Planck plus existing galaxy surveys can reach $\gtrsim 14\sigma$ detection. Existing CMB-LSS cross correlation measurements shall be reanalyzed to test the existence of the dark flow and, if it exists, shall be used to eliminate possible bias on the integrated Sachs-Wolfe effect measurement through the CMB-LSS cross correlation.

Key words: (cosmology:) large-scale structure of Universe: cosmic microwave background: theory: observations

1 INTRODUCTION

Recently, Kashlinsky et al. (2008, 2009); Atrio-Barandela et al. (2010); Kashlinsky et al. (2010) analyzed CMB fluctuations on directions of X-ray galaxy clusters and found a large bulk flow with speed $v_{\rm DF} \sim 10^3$ km/s to $z_{\rm DF} \sim 0.3$, toward direction $(l_0, b_0) \sim (290°, 30°)$. This extraordinarily large bulk flow (the so called dark flow) is a severe challenge to the standard ΛCDM paradigm and has fundamental implications on the topology of the universe, the inflationary scenario and the nature of gravity (e.g. Carroll et al. 2008; Afshordi et al. 2009; Chang et al. 2009; Mersini-Houghton & Holman 2009; Khoury & Wyman 2009; Kashlinsky et al. 2010 and references therein).

The above dark flow measurements are under scrutiny (Keisler 2009; Atrio-Barandela et al. 2010). It also requires independent confirmations, from direct (Feldman et al. 2009; Watkins et al. 2009) and indirect (Lavaux et al. 2010) velocity measurements of nearby galaxies and nearby supernovae (Gordon et al. 2008). In this paper, we point out that, besides the dipole from which the dark flow is inferred, the dark flow also induces small angular scale CMB tempera-

ture fluctuations. If the amplitude of the dark flow reaches the reported value of $\sim 10^3$ km/s, it shall be detected by combining existing CMB and galaxy measurements.

If the matter distribution where the dark flow resides is homogeneous, the dark flow only induces a CMB dipole. However, from galaxy surveys, we know that the local universe is strongly inhomogeneous in the density distribution (e.g. Tegmark et al. 2004). Given these inhomogeneities, even an uniform dark flow can induce small angular scale temperature fluctuations in the CMB sky, through exactly the same mechanism of the inverse Compton scattering to generate the conventional kinetic Sunyaev Zel'dovich (SZ) effect (Sunyaev & Zeldovich 1972, 1980),

$$
\begin{aligned}
\Theta_{\rm DF}(\hat{n}) &\equiv \frac{\Delta T_{\rm DF}(\hat{n})}{T_{\rm CMB}} = \int n_e(\hat{n}, z)\sigma_T a d\chi \frac{\mathbf{v}_{\rm DF} \cdot \hat{n}}{c} \\
&= \left[6.1 \times 10^{-6} \frac{v_{\rm DF}}{10^3 {\rm km}/s} \int_0^{z_{\rm DF}} (1+z)^2 d\tilde{\chi} \right] \cos\theta \\
&+ 6.1 \times 10^{-6} \frac{v_{\rm DF}}{10^3 {\rm km}/s} \cos\theta \int_0^{z_{\rm DF}} \delta_e(\hat{n}, z)(1+z)^2 d\tilde{\chi}
\end{aligned}
\tag{1}
$$

Here, $\cos\theta \equiv \hat{n}_{\rm DF} \cdot \hat{n}$ is the cosine between the dark flow direction $\hat{n}_{\rm DF}$ and the direction $\hat{n}$. $z_{\rm DF}$ is the edge of the

dark flow. $n_e = \bar{n}_e(1 + \delta_e)$ is the 3D free electron number density, $\bar{n}_e$ is the mean number density and δ_e is the over-density. Throughout the paper we adopt the fiducial value $\Omega_b h = 0.031$ and thus neglect the prefactor $\Omega_b h/0.031$ in Eq. 1. $\tilde{\chi} \equiv \chi/(c/H_0)$ is the dimensionless comoving distance in unit of the Hubble radius c/H_0. Throughout the paper we adopt a flat ΛCDM cosmology with $\Omega_m = 0.27$, $\Omega_\Lambda = 1 - \Omega_m$, $\Omega_b = 0.044$, $\sigma_8 = 0.84$ and $h = 0.71$.

The first term in the last expression of Eq. 1 is a dipole term. However, the second term is not, despite its dipole-like prefactor $\cos \theta$. The electron over-density $\delta_e(\hat{n})$ has a complicated directional dependence and is clustered at scales $\lesssim 100\ h^{-1}$ Mpc. It is this density modulation from all free electrons producing the small scale kinetic SZ effect, the one that we point out and investigate in this paper. It clearly differs from the CMB dipole induced by the Earth motion and the CMB dipole induced by the dark flow of uniform electron distribution, both are lacking of the δ_e modulation. It also differs from the conventional kinetic SZ effect, which is further modulated by the non-uniform velocity (Vishniac 1987). Later we will find that the two kinetic SZ effects have different clustering behaviors for this reason.

This dark flow induced small scale kinetic SZ effect leaves unique imprints on the CMB sky. We will estimate its auto power spectrum and its correlation with the large scale structure (LSS). The exact value of $v_{\rm DF}$ is highly uncertain (Kashlinsky et al. 2010). Throughout the paper, we will adopt a fiducial value $v_{\rm DF} = 10^3$ km/s. The results presented in this paper can be scaled to other value of $v_{\rm DF}$ straightforwardly. Given $v_{\rm DF} = 10^3$ km/s, we find that it is promising to extract the dark flow component through CMB-LSS correlations in an unbiased manner, if the CMB maps are properly weighted with a directional dependent weighting factor. We estimate that existing data may already allow for $\gtrsim 6\sigma$ detection of this effect and provide independent test on the dark flow scenario.

2 THE AUTO POWER SPECTRUM OF THE DARK FLOW INDUCED KINETIC SZ EFFECT

The dark flow induced kinetic SZ temperature fluctuation is correlated at small scales due to clustering of the underlying electron overdensity δ_e. We derive the resulting auto power spectrum averaged over the survey area (the appendix §A),

$$
\begin{aligned}
\frac{\ell^2 C_{\rm DF}(\ell)}{2\pi} &= 3.7 \times 10^{-11} \left[\frac{v_{\rm DF}}{10^3 {\rm km}/s} \right]^2 f_T \\
&\times \frac{\pi}{\ell} \int_0^{z_{\rm cut}} \Delta_e^2 (k = \frac{\ell}{\chi}, z)(1+z)^4 \tilde{\chi} d\tilde{\chi} \ . \quad (2)
\end{aligned}
$$

The expression in the integral has adopted the well known Limber approximation. $\Delta_e^2(k, z)$ is the electron number density power spectrum (variance) at scale k and redshift z. Throughout the paper we approximate it as $\Delta_e^2 = \Delta_m^2$. Here Δ_m^2 is the matter power spectrum (variance). Since the measured matter distribution in the nearby universe agrees with the standard ΛCDM (Tegmark et al. 2004), it allows us to adopt the CMBFAST transfer function (Seljak & Zaldarriaga 1996). The nonlinear power spectrum Δ_m^2 is calculated by the halofit formula (Smith et al. 2003).

The suppression factor f_T is

$$
f_T = \left\langle \cos^2 \theta \right\rangle_S \ . \quad (3)
$$

The average is over all directions in the survey area. Clearly f_T is a direction dependent quantity and varies from survey to survey. Namely, the dark flow induced kinetic SZ effect is statistically anisotropic, unlike other isotropic CMB components. Eq. 3 is derived under the small angle approximation. Its accuracy is of the order $(\Delta\theta)^2/4 \sim (\pi/\ell)^2 \ll 1$ when $\ell \geqslant 10$, where $\Delta\theta$ is the typical angular scale involved, $\Delta\theta \sim 2\pi/\ell$. Refer to the appendix §A for more details. For a full sky survey $f_T = 1/3$.

This characteristic directional dependence can in principle be applied to separate the dark flow induced kinetic SZ effect from isotropic components (primary CMB, the thermal SZ effect, the conventional kinetic SZ effect, etc.) in the auto correlation measurement. Alternatively one can weigh the CMB temperature fluctuations by a directional dependent function $W(\hat{n})$. It can be desired to amplify the dark flow component with respect to other components. In this case,

$$
f_T = \langle \cos^2 \theta W^2(\hat{n}) \rangle_S \ . \quad (4)
$$

$C_{\rm DF}(\ell)$ shows a number of difference in the ℓ dependence comparing to that of the conventional kinetic SZ effect (Fig. 1), the major reason is that the dark flow induced kinetic SZ fluctuation is linear in density fluctuations while the conventional one is quadratic through the interplay between density and velocity inhomogeneity. Another reason is that the dark flow induced kinetic SZ effect comes from much lower redshift than the conventional kinetic SZ effect. Since the typical redshift of the dark flow induced kinetic SZ effect is proportional to $z_{\rm DF}$, the peak moves to smaller scale (larger ℓ) when $z_{\rm DF}$ increases. For the fiducial value of $v_{\rm DF} = 10^3$ km/s out to $z_{\rm DF} = 0.3$, the dark flow induced kSZ power spectrum, averaged over the whole sky with $W = 1$, peaks at $\ell \simeq 2000$ with an amplitude $1.6\mu{\rm K}^2$, a factor of ~ 2 smaller than the conventional kinetic SZ effect (e.g. Ma & Fry 2002; Zhang et al. 2004; Hernández-Monteagudo & Ho 2009) at the same scale. Overall, this dark flow induced kinetic SZ effect is non-negligible, ranging from $\sim 100\%$ of the conventional one at $\ell = 200$ to $\sim 30\%$ at $\ell = 10^4$.

The SPT collaboration measured a combined SZ power spectrum (tSZ+0.46× kSZ) at 150 GHz band and $\ell = 3000$ to be $4.2 \pm 1.5\mu{\rm K}^2$ (Lueker et al. 2009). This measurement is a factor of ~ 2 lower than the predicted thermal SZ effect with $\sigma_8 = 0.8$ (e.g. Zhang et al. 2002; Lueker et al. 2009), but consistent with recent WMAP and ACT measurements (Komatsu et al. 2010; The ACT Collaboration et al. 2010). The dark flow induced kinetic SZ effect is $\sim 0.3\mu{\rm K}^2$ at the analyzed SZ sky where $f_T \simeq 0.14$. Given its sub-dominance to the conventional kinetic SZ effect and the thermal SZ effect (Fig. 1), it is difficult to perform a robust test of the reported dark flow against existing SZ power spectrum measurements. Future large-area SZ measurements at 217 GHz, the null of the thermal SZ effect, may be able to infer its existence through the directional dependence in the f_T prefactor.
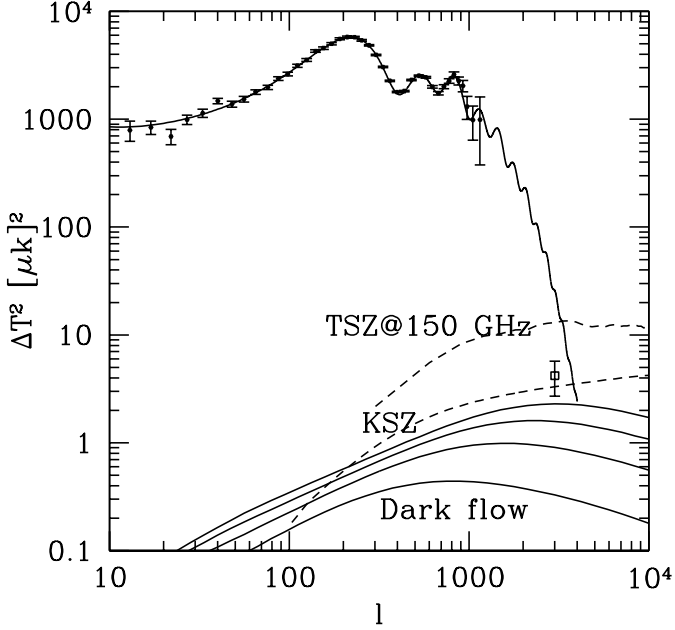
Figure 1. The auto correlation power spectrum of the dark flow induced kinetic SZ effect. The primary CMB data points are the seven years WMAP result (Komatsu et al. 2010; Larson et al. 2010). The SZ measurement at $\ell = 3000$ is from Lueker et al. (2009). The thermal SZ result is the one in Zhang et al. (2002), scaled from the original $\sigma_8 = 1.0$ to $\sigma_8 = 0.8$ with the σ_8^7 scaling and scaled from the Raleigh-Jeans regime to 150 GHz. The kinetic SZ result is the homogeneous kinetic SZ power spectrum predicted from the model of Zhang et al. (2004). The four solid lines are for the dark flow models with $z_{\rm DF} = 0.1, 0.2, 0.3, 0.4$ from bottom up. We have adopted $v_{\rm DF} = 10^3$ km/s. The shown power spectra is for a full sky survey, with the suppression factor $f_T = 1/3$. The SPT sky (Lueker et al. 2009) has $f_T \sim 0.14$, so the dark flow contribution to the measured SZ effect at $\ell = 3000$ is $\sim 0.3\mu{\rm K}^2$, sub-dominant to other components.

3   CROSS CORRELATION WITH GALAXIES

The dark flow induced kinetic SZ effect is also correlated with tracers of the large scale structure at $z < z_{\rm DF}$. The resulting cross power spectrum between the dark flow induced kinetic SZ effect and the galaxy distribution is

$$\frac{\ell^2 C_{Tg}}{2\pi} \simeq 6.1 \times 10^{-6} \frac{v_{\rm DF}}{10^3 {\rm km}/s} f_{Tg} \tag{5}$$

$$\times \quad \frac{\pi}{\ell} \int_0^{z_{\rm cut}} \Delta_{eg}^2 (k = \frac{\ell}{\chi}, z)(1+z)^2 \tilde{\chi} \bar{n}_g(z) dz \ .$$

$\bar{n}_g$ is the galaxy distribution function normalized that $\int \bar{n}_g(z) dz = 1$. We approximate the electron-galaxy cross power spectrum $\Delta_{eg}^2 = b_g \Delta_m^2$ where b_g is the galaxy bias. The suppression factor

$$f_{Tg} = \langle \cos\theta W(\hat{n}) \rangle_S \ . \tag{6}$$

Here the average is over the overlapping sky of CMB and galaxy surveys. f_{Tg} is also direction dependent and varies from survey to survey. Especially, $f_{Tg} = 0$ with $W = 1$ for full sky surveys. Thus for WMAP+2MASS, we have to choose $W \neq 1$ to avoid the cancellation. A natural choice is $W(\hat{n}) = \cos\theta$, for which $f_{Tg} = 1/3$. A nice property about
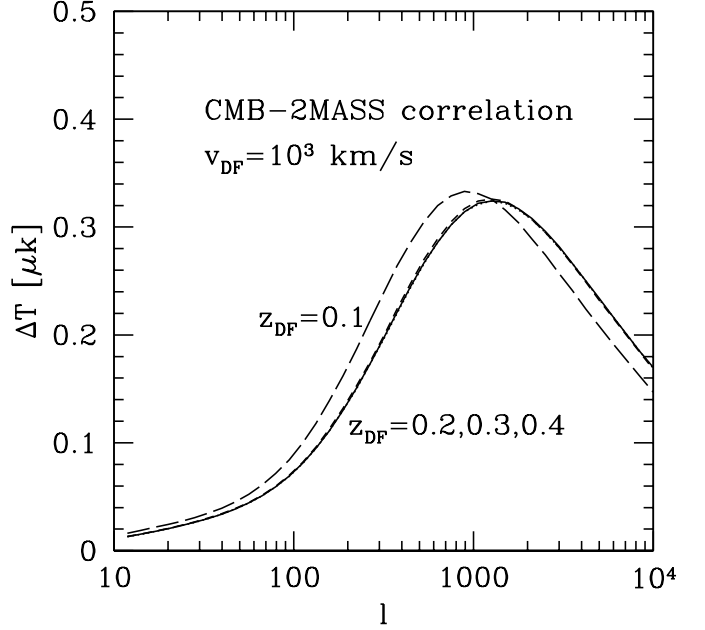

Figure 2. The cross correlation power spectrum between the dark flow induced kinetic SZ effect and a full sky galaxy survey of 2MASS-like. A directional dependent weighting $W = \cos\theta$ is adopted. Since most 2MASS galaxies reside at $z < 0.1$, the cross correlation varies little for bulk flows beyond $z \gtrsim 0.1$. For this reason, the three curves of $z_{\rm DF} = 0.2, 0.3, 0.4$ are barely distinguishable.

this choice is that $\langle W \rangle_S = 0$ and thus isotropic components in CMB maps (primary CMB, the thermal SZ, conventional kinetic SZ effect, dusty star forming galaxies, etc.) do not bias the cross correlation.[1]

The cross correlation signal between the dark flow induced kinetic SZ effect and 2MASS galaxy distribution is shown in Fig. 2. The galaxy distribution is adopted from Afshordi et al. (2004), along with the galaxy bias $b_g = 1.18$. Since the galaxy distribution peaks at $z < 0.1$, the peak of the cross power spectrum is at lower ℓ than that in the auto power spectrum. For a dark flow with $v_{\rm DF} = 10^3$ km/s out to $z_{\rm DF} = 0.3$, the cross power spectrum peaks at $\ell \simeq 1.3 \times 10^3$ with amplitude $0.32\mu{\rm K}$.

Given the existence of this cross correlation, it is interesting to ask whether it will impact the interpretation of existing CMB-LSS cross correlation measurements, whose primary goal is to detect the integrated Sachs-Wolfe effect (ISW) (Giannantonio et al. (2008) and references therein). Since $f_{Tg} = 0$ with $W = 1$ for full sky surveys, this dark flow induced kinetic SZ effect is unlikely to significantly bias the integrated Sachs-Wolfe (ISW) measurement through WMAP+2MASS (NVSS), unless the CMB masks causes significant deviation of $f_{Tg} \neq 0$. For surveys with partial sky coverage, $f_{Tg} \neq 0$ in general. So the ISW mea-

---

[1] Realistic correlation analysis masked the galactic plane, so $f_{Tg} \neq 0$ even for full sky surveys like WMAP+2MASS. This issue is important for actual data analysis. But for the theoretical study presented in this paper it is safe to neglect it.
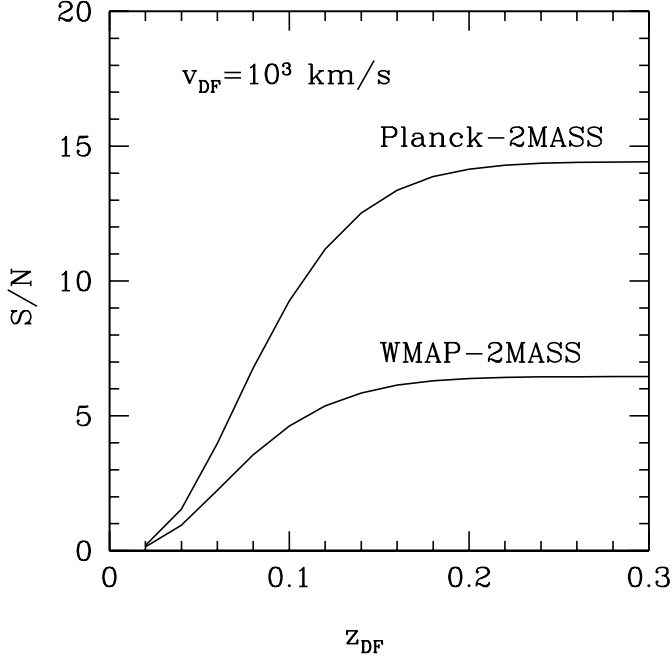
Figure 3. The predicted S/N of WMAP-2MASS (Planck+2MASS) cross correlation designed for the dark flow detection, for which an unbiased optimal directional dependent weighing scheme is applied. WMAP+2MASS is able to detect the bulk flow at $\sim 6\sigma$ confidence level. Planck+2MASS is able to improve the S/N by a factor of ~ 2. Since most 2MASS galaxies reside below $z = 0.1$, it is inefficient to detect the bulk flow beyond $z = 0.1$, explaining the plateau in the S/N curves. Cross correlating WMAP and Planck with deeper galaxy surveys such as SDSS, LAMOST and BOSS will improve the detection at $z > 0.1$. Thus existing surveys (WMAP, 2MASS and SDSS especially) are already able to detect the dark flow reported by Kashlinsky et al. (2008, 2010). Since the S/N $\propto v_{\rm DF}$, the cross correlation measurement of existing surveys can detect dark flow with speed ~ 500 km/s at $\gtrsim 3\sigma$ confidence level and Planck+2MASS can improve the sensitivity to ~ 200 km/s.

surements based on these surveys are biased by this component. Interestingly, depending on the direction of survey area, this dark flow induced kinetic SZ effect may increase or decrease the measured correlation strength. Given the comparable correlation strength of the two effects, this issue shall be taken into account of future data analysis to avoid bias in dark energy constraint, especially for low redshift galaxy surveys with partial sky coverage like SDSS.

Thanks to its characteristic directional dependence, at least in principle we are able to measure the dark flow through CMB-LSS cross correlation in a way unbiased by other CMB components such as the ISW effect. To do so, we need to design a weighting scheme such that $\langle W \rangle_S = 0$ to eliminate isotropic components in CMB maps (primary CMB, the thermal SZ, conventional kinetic SZ effect, dusty star forming galaxies, the ISW effect, etc.). The statistical measurement error under such weighting is

$$\frac{\Delta C_{Tg}}{C_{Tg}} = \sqrt{\frac{1 + \langle W^2 \rangle_S (C^{\rm CMB} + C^{\rm CMB,N})(C_g + C_{g,N})/C_{Tg}^2}{2\ell \Delta\ell f_{\rm sky}}} \quad (7)$$

Here, $C^{\rm CMB}$ is the power spectrum of primary CMB and $C^{\rm CMB,N}$ is the one of other components including the instrumental noise. Since $C_{\rm DF} \ll C^{\rm CMB} + C^{\rm kSZ} + \cdots$, we can safely neglect the contribution of $C_{\rm DF}$ in $C^{\rm CMB,N}$. C_g is the galaxy power spectrum and $C_{g,N}$ is the associated measurement noise, $C_{g,N} = 4\pi f_{\rm sky}/N_g$ where N_g is the total number of galaxies. $f_{\rm sky}$ is the fractional sky coverage.

We not only want the weighting to be unbiased ($\langle W \rangle_S = 0$), but also want it to be optimal such that the measurement error is minimized. It turns out the unbiased optimal weighting is the solution to a 2nd integral equation derived in the appendix §B.

The forecast for WMAP(PLANCK)-2MASS cross correlation measurement is shown in Fig. 3. We take the impact of galactic mask into account and adopt $f_{\rm sky} = 0.7$ to evaluate the cosmic variance and shot noise. The unbiased optimal weighting W for this survey configuration is not trivial to derive. However, for the error estimation presented here, it is safe to adopt the one corresponding to the case of full sky coverage, for which we have the analytical solution $W(\hat{n}) = \cos\theta$. Under such weighting we have $\langle W^2 \rangle_S = f_{Tg} = 1/3$. We find that WMAP plus 2MASS is able to measure the dark flow, if it extends to $z \geqslant 0.1$ with an amplitude 10^3 km/s, at $\sim 6\sigma$. Due to the resolution of WMAP, it will miss the peak of the correlation at $\ell \sim 10^3$. On the other hand, Planck, with better angular resolution, will be able to well capture the peak correlation and is thus able to improve the measurement to $\sim 14\sigma$.

Since most 2MASS galaxies locate at $z < 0.1$, 2MASS is inefficient to probe the dark flow at $z > 0.1$. This explains the S/N plateau at $z_{\rm DF} > 0.1$ (Fig. 3). Deeper surveys such as SDSS are more suitable for this purpose and can improve the overall S/N significantly. We expect that combining WMAP and existing galaxy surveys are already able to detect the dark flow at $z \lesssim 0.3$ with the claimed amplitude and depth at $\gtrsim 6\sigma$ level. Planck will further improve it to $\gtrsim 14\sigma$, for $v_{\rm DF} = 10^3$ km/s. Since the S/N $\propto v_{\rm DF}$ in the cross correlation measurement, this implies that, Planck plus existing galaxy surveys are able to detect dark flow with amplitude as low as 200 km/s at $\gtrsim 3\sigma$.

4   SUMMARY

We have pointed out the existence of the dark flow induced small scale kinetic SZ effect and estimated its amplitude. As a potentially non-negligible component of CMB temperature fluctuations, it impacts cosmology in at least two ways.

• It enables a useful independent check on existing dark flow measurements (Kashlinsky et al. 2008, 2009; Atrio-Barandela et al. 2010; Kashlinsky et al. 2010). The direction-weighted CMB-LSS cross correlation measurement proposed in this paper should be able to detect a dark flow with an amplitude of $\gtrsim 500$ km/s at $\gtrsim 3\sigma$ level for WMAP+2MASS. Planck+2MASS can improve the detection threshold to 200 km/s. Given this sensitivity, it will allow for a strong test of the existence of the dark flow.

• A dark flow of the reported amplitude $\sim 10^3$ km/s can significantly bias the ISW measurement through the CMB-LSS cross correlation, if the survey area is sufficiently close to the direction of the dark flow or the opposite of it. Hence it can significantly bias the dark energy constraint based on

the ISW interpretation. Existing data shall be reinterpreted to avoid such bias.

REFERENCES

The ACT Collaboration, et al. 2010, arXiv:1001.2934

Afshordi, N., Loh, Y.-S., & Strauss, M. A. 2004, PRD, 69, 083524

Afshordi, N., Geshnizjani, G., & Khoury, J. 2009, Journal of Cosmology and Astro-Particle Physics, 8, 30

Atrio-Barandela, F., Kashlinsky, A., Ebeling, H., Kocevski, D., & Edge, A. 2010, arXiv:1001.1261

Carroll, S. M., Tseng, C.-Y., & Wise, M. B. 2008, arXiv:0811.1086

Chang, S., Kleban, M., & Levi, T. S. 2009, Journal of Cosmology and Astro-Particle Physics, 4, 25

Feldman, H. A., Watkins, R., & Hudson, M. J. 2009, arXiv:0911.5516

Giannantonio, T., Scranton, R., Crittenden, R. G., Nichol, R. C., Boughn, S. P., Myers, A. D., & Richards, G. T. 2008, PRD, 77, 123520

Gordon, C., Land, K., & Slosar, A. 2008, MNRAS, 387, 371

Hernández-Monteagudo, C., & Ho, S. 2009, MNRAS, 398, 790

Kashlinsky, A., Atrio-Barandela, F., Kocevski, D., & Ebeling, H. 2008, ApJ, 686, L49

Kashlinsky, A., Atrio-Barandela, F., Kocevski, D., & Ebeling, H. 2009, ApJ, 691, 1479

Kashlinsky, A., Atrio-Barandela, F., Ebeling, H., Edge, A., & Kocevski, D. 2010, ApJ, 712, L81

Keisler, R. 2009, ApJ, 707, L42

Khoury, J., & Wyman, M. 2009, PRD, 80, 064023

Komatsu, E., et al. 2010, arXiv:1001.4538

Larson, D., et al. 2010, arXiv:1001.4635

Lavaux, G., Tully, R. B., Mohayaee, R., & Colombi, S. 2010, ApJ, 709, 483

Lueker, M., et al. 2009, arXiv:0912.4317

Ma, C.-P., & Fry, J. N. 2002, Physical Review Letters, 88, 211301

Mersini-Houghton, L., & Holman, R. 2009, Journal of Cosmology and Astro-Particle Physics, 2, 6

Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437

Smith, R. E., et al. 2003, MNRAS, 341, 1311

Sunyaev, R. A., & Zeldovich, Y. B. 1972, Comments on Astrophysics and Space Physics, 4, 173

Sunyaev, R. A., & Zeldovich, I. B. 1980, MNRAS, 190, 413

Tegmark, M., et al. 2004, ApJ, 606, 702

Vishniac, E. T. 1987, ApJ, 322, 597

Watkins, R., Feldman, H. A., & Hudson, M. J. 2009, MNRAS, 392, 743

Zhang, P., Pen, U.-L., & Wang, B. 2002, ApJ, 577, 555

Zhang, P., Pen, U.-L., & Trac, H. 2004, MNRAS, 347, 1224

APPENDIX A: DERIVING THE SUPPRESSION FACTOR

We first subtract the dipole mode in Eq. 1 and rewrite the rest as

$$\Theta_{\rm DF} = \cos\theta \times y(\hat{n}) \; ; \; y(\hat{n}) \propto \int \delta_e (1+z)^2 d\tilde{x} \; . \tag{A1}$$

The expectation value of the angular correlation between two fixed directions $\hat{n}_1$ and $\hat{n}_2$ is

$$
\begin{aligned}
\langle \Theta_{\rm DF}(\hat{n}_1)\Theta_{\rm DF}(\hat{n}_2)\rangle &= \cos\theta_1 \cos\theta_2 \langle y(\hat{n_1})y(\hat{n_2})\rangle \tag{A2}\\
&= \left[\cos^2\theta + O(\frac{\theta_{12}^2}{4})\right]\langle y(\hat{n}_1)y(\hat{n}_2)\rangle \; .
\end{aligned}
$$

Here, $\theta \equiv (\theta_1 + \theta_2)/2$ and $\theta_{12} \equiv \theta_1 - \theta_2$. For small angular separation $|\Delta\theta| \ll 1$ ($\hat{n}_1 \cdot \hat{n}_2 \equiv \cos\Delta\theta$), we have $|\theta_{12}| \leqslant |\Delta\theta| \ll 1$. So the term $\theta_{12}^2/4 \ll 1$ and can be safely neglected. Averaging over all pairs with the same $\Delta\theta$ in the survey volume and using the fact that $\langle y(\hat{n}_1)y(\hat{n}_2)\rangle$ should be isotropic, we have

$$\langle \Theta_{\rm DF}(\hat{n}_1)\Theta_{\rm DF}(\hat{n}_2)\rangle_S \simeq \langle\cos^2\theta\rangle_S \langle y(\hat{n}_1)y(\hat{n}_2)\rangle \; .$$

Fourier transforming the above equation, we obtain Eq. 2 & Eq. 3 and recognize $f_T = \langle\cos^2\theta\rangle_S$ for the case of $W = 1$. For a general W, $f_T = \langle\cos^2\theta W^2\rangle_S$ can be derived following the same procedure.

APPENDIX B: THE UNBIASED OPTIMAL WEIGHTING FUNCTION

The optimal weighting $W(\hat{n})$ shall minimize $\langle W_S^2\rangle/f_{Tg}^2 \propto \langle W^2\rangle_S/\langle\cos\theta W\rangle_S^2$, under the constraint $\langle W\rangle_S = 0$. By the Lagrange multiplier method, this corresponds to minimize

$$\frac{\langle W^2\rangle_S}{\langle\cos\theta W\rangle_S^2} - \lambda\langle W\rangle_S \; . \tag{B1}$$

This requires

$$W = \frac{\langle W^2\rangle_S}{\langle\cos\theta W\rangle_S}\left[\cos\theta - \langle\cos\theta\rangle_S\right] \; . \tag{B2}$$

For a general configuration of sky survey area, it is nontrivial to solve the above second order integral equation. However, for the full sky coverage, since $\langle\cos\theta\rangle_S = 0$, one can easily find the solution to be $W = \cos\theta \equiv \hat{n}_{\rm DF} \cdot \hat{n}$. This is what we adopt to estimate the expected WMAP+2MASS cross correlation S/N.